Modelling the geodynamo: progress and challenges

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It is widely accepted that the Earth’s magnetic field is powered by a convection-driven dynamo operating in its liquid iron core. The twentieth century witnessed remarkable advances in the field of magnetohydrodynamics, which eventually led to three-dimensional computer simulations of the geodynamo. In this review we look at the significant developments that shaped our present understanding of magnetic field generation in the Earth’s core. We also examine the successes and shortcomings of current geodynamo models.

Keywords: Earth’s core, geodynamo, geomagnetism, magnetohydrodynamics.

Introduction

The Earth has a large-scale dipolar magnetic field, a fact of historical importance because of the role of the magnetic compass in the exploration of our planet. The magnetic lines of force originate from the magnetic North and South Poles, which are presently about 11.5° away from the geographic North and South Poles. The Earth’s magnetic field acts as a shield against high-energy particles from the Sun and outer space, thereby protecting our atmosphere and the life that it supports. No better reason can be given for understanding the Earth’s magnetic field and its evolution over 4 billion years. The Earth’s field has varied considerably over geological time, sometimes being weak, sometimes strong and intermittently reversing direction completely, so that North becomes South and South becomes North. This pattern of changes, and notably the polarity flips, have left a distinctive fingerprint on the surface of the Earth. Palaeomagnetists examine rocks and seabed sediments which formed in ancient times to follow the long-time behaviour of the geomagnetic field. For instance, magnetic minerals crystallize in cooling lava flows and orientate themselves towards the magnetic North Pole. This magnetic record is permanently locked in the rocks when they harden. Data from volcanic rocks and sediments show that the last flip in magnetic field polarity occurred about 780,000 years ago. Direct vector measurements of the geomagnetic field were pioneered by Carl Friedrich Gauss in the 1830s, and since the 1960s an excellent global distribution of the field has been provided by satellites. However, at length scales shorter than 2600 km the core magnetic field is obscured by the remnant crustal field, thus limiting our knowledge of the field in the planet’s deep interior.

The concept of magnetic field generation goes back to Michael Faraday, who showed that an electrical conductor moving in a static magnetic field produces an electric current. This was the principle behind his disk dynamo, which consisted of a conducting disk spinning in a magnetic field. The next step was to examine whether this induced electric current could, in turn, produce a magnetic field that reinforces the original field. A disk dynamo can be designed such that the induced electric current flows through a loop in the same direction as the sense of spin; this results in an induced magnetic field that points in the same direction as the pre-existing field. It was Larmor who first suggested that an electrically conducting fluid in which suitable motions were produced could sustain magnetic fields in the Sun and Earth. Earlier studies in seismology had already led to the inference that the Earth’s outer core is liquid because of its inability to transmit transverse (shear) waves. Hence Larmor’s idea of a self-excited fluid dynamo was an attractive proposition for the Earth. Why do we need a dynamo theory for the Earth? If there were no fluid motions in the core, any primordial magnetic field would have decayed away on a timescale of ~ 10^4 years. Yet, the Earth has had a magnetic field for ~ 10^9 years, which can be explained by a process of field generation through induction in its core. Fluid motion in the outer core is thought to be driven either by natural convection or by buoyant plumes of light material released from the boundary of the inner core as pure iron crystallizes. The presence of dissolved radioactive heat sources cannot be ruled out. The iron-rich core ensures that the motion of the fluid in a magnetic field is an inductive process that generates new magnetic field through stretching and twisting of flux tubes by the background velocity, the process being limited by magnetic diffusion. Alternative mechanisms for generation of the Earth’s magnetic field, such as thermoelectric and electrochemical effects, have been proposed, but they cannot plausibly provide the energy required to maintain the observed field.
Although observation of the Earth’s magnetic field has a long history of over 400 years\(^6\), geodynamo theory made significant progress only in the last century due to advances in the subject of magnetohydrodynamics (MHD), which deals with the flow of electrically conducting fluids in magnetic fields. The development of numerical methods and solutions and the advent of fast computers aided this progress. The self-consistent dynamo problem requires solution of the MHD equations, which simultaneously determine the magnetic field, velocity and temperature (or composition) in a conducting fluid. Recent advances in computational ability have enabled us to perform three-dimensional simulations of the geodynamo, which provide realizations of geomagnetic field features such as the dipolar structure, secular variation (time-changes of the magnetic field), high-latitude magnetic flux concentrations and polarity reversals. The aim of this article is to discuss the progress made over the decades in modelling the geodynamo and the challenges that lie ahead. This review is by no means exhaustive; aspects of the geodynamo not covered here can be found in earlier reviews\(^7\)–\(^11\).

**Early developments in geodynamo theory**

It was natural for early investigators to consider rotating MHD systems in which both the velocity and magnetic fields were axisymmetric. As the Earth’s external field is essentially a dipole, one might look at a steady, axisymmetric dynamo in which the magnetic field \(B\) is poloidal, \((B_r, 0, B_z)\) in cylindrical polar coordinates \((r, \theta, z)\). The electric current \(j\) is then toroidal, \((0, j_\theta, 0)\). The velocity field \(u\) is also poloidal. Cowling\(^12\) considered such an idealized system and concluded that an axisymmetric magnetic field could not be supported by axisymmetric fluid motions. His argument was that an axisymmetric poloidal field always has a neutral ring where \(u\) is zero. This anti-dynamo theorem showed that nonaxisymmetric configurations had to be considered to make progress in dynamo theory. (It was, however, shown later that axisymmetric flows could support non-axisymmetric fields.) It was Elsasser\(^13\) who initiated the study of the interaction between non-axisymmetric (three-dimensional) velocity and magnetic fields. He also suggested decomposing the two fields into poloidal and toroidal components and then expanding them in spherical harmonics. This approach was developed further by Bullard and Gellman\(^14\) and is being used in dynamo models today. For instance,

\[
\mathbf{u} = \nabla \times (\mathbf{Tr}) + \nabla \times \nabla \times (\mathbf{Pr});
\]

where \(T\) and \(P\) are the toroidal and poloidal components of \(\mathbf{u}\) and \(Y_l^m\) is a normalized spherical harmonic function.

Bullard and Gellman outlined a cyclic process by which a poloidal magnetic field can regenerate itself (see pages 259–260 of their paper)\(^15\). A toroidal field is swept out from an existing poloidal field through differential rotation (Figure 1\(a\)); and an upwelling followed by a twist recreates a poloidal field from a toroidal field (Figure 1\(b\)). These two events came to be known as the \(\alpha\)-effect and the \(\alpha\)-effect respectively. The concept of the \(\alpha\)-effect was developed further by Parker\(^15\), who suggested that the deformation of the toroidal field can happen in cyclones and anticyclones similar in structure to those found in the atmosphere. Steenbeck et al.\(^16\) provided a mathematical framework for the \(\alpha\)-effect by noting that a small-scale, non-axisymmetric velocity \(\mathbf{u}\) interacts with a small-scale magnetic field \(\mathbf{b}'\) to generate a large-scale electromotive force \(\mathbf{E} = \mathbf{u} \times \mathbf{b}'\), which, in turn, is proportional to the mean magnetic field \(\mathbf{B}_\circ\). (The constant of proportionality here is denoted by \(\alpha\).) The small-scale motion can be generated in the Earth’s core either by free convection or by buoyant blobs of light elements released from a mushy zone near the inner core boundary\(^7\).

The popularity of the \(\alpha\)-effect inevitably led to the kinematic dynamo problem\(^17\)–\(^20\), which addresses the question of whether a given flow can generate a magnetic field or not. The magnetic field is governed by Maxwell’s equations and Ohm’s law for a moving conductor\(^21\). Combining these gives the magnetic induction equation, which determines the evolution of \(B\):

\[
T(r, \theta, \phi, t) = \sum_{l=0}^{L} \sum_{m=-l}^{l} T_l^m(r, t) Y_l^m(\theta, \phi),
\]

**Figure 1.** Schematic of the \(\alpha\)-\(\alpha\) dynamo cycle\(^1\)\(^10\)\(^14\). \(a\), An initial poloidal field is swept by differential rotation to give a toroidal field; \(b\), Fluid motion lifts and twists a toroidal field line to produce a poloidal field loop.
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \]  
(3)

where \( \eta \) is the magnetic diffusivity. Magnetic field growth happens when convection of \( \mathbf{B} \), given by the first term on the right-hand side, exceeds magnetic diffusion, given by the second term. The ratio of the two terms gives the magnetic Reynolds number, \( R_m = \frac{u_s L}{\eta} \), where \( u_s \) is the typical velocity and \( L \) is the lengthscale.

Although kinematic dynamos have been successful in telling us which flows can produce magnetic fields resembling that of the Earth, they have ignored the effect of the magnetic field on the velocity. To ensure the coupled evolution of \( \mathbf{u} \) and \( \mathbf{B} \), the induction equation (3) must be solved in conjunction with the momentum equation for a liquid metal. And if the flow is driven by, say, thermal convection, then the temperature must also be solved for.

We therefore have the additional equations,

\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + 2 \mathbf{\Omega} \times \mathbf{u} = -\frac{1}{\rho_0} \nabla p + \frac{\rho}{\rho_0} g \nabla z + \frac{1}{\rho_0 \mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nu \nabla^2 \mathbf{u}, \]

(4)

\[ \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \kappa \nabla^2 T + Q_s, \]

(5)

where \( \mathbf{u} \) and \( \mathbf{B} \) also satisfy the divergence-free conditions

\[ \nabla \cdot \mathbf{u} = 0; \quad \nabla \cdot \mathbf{B} = 0. \]

(6)

The terms on the left-hand side of eq. (4) represent linear and nonlinear inertia (which give the material derivative \( Du/Dt \)), and the Coriolis force. The forces on the right-hand side are, in order of appearance, the fluid pressure modified by centrifugal acceleration, buoyancy, magnetic (Lorentz) force and viscous diffusion. In the above equation, \( \mu_0 \) is the permeability of free space, \((1/\mu_0) \nabla \times \mathbf{B}\) the current density \( j \) by Ampere’s law, \( \mathbf{\Omega} \) the background rotation vector that points in the \( z \)-direction, \( g \) the local gravity pointing downward, \( \rho \) and \( \rho_0 \) the local and far-field densities, \( \kappa \) the thermal diffusivity and \( Q_s \), a uniform volumetric heat source/sink.

Before discussing the solutions of the MHD eqs (3)–(6), a note on convection subject to rotation and magnetic field is appropriate.

**Onset of convection and the effects of rotation and magnetic field**

The classical problem of *Rayleigh–Bénard convection* consists of a fluid layer confined between two plates of infinite horizontal extent and heated from below. The difference in temperature across the layer, \( \Delta T \) is related to the difference in density, \( \Delta \rho \) via the Boussinesq approximation, which gives \( \Delta \rho = -\beta \Delta T \), where \( \beta \) is the volumetric expansion coefficient and \( \rho_0 \) is the density at the upper boundary, where the temperature is \( T_0 \). As the temperature difference across the layer exceeds a critical value, up-and-down convective motions are set up. The driving force for these motions is buoyancy, which is the difference between the force of gravity acting on light and heavy fluid elements. Now, the effect of background rotation on this fluid layer may be understood by looking at the curl of the momentum conservation eq. (4) for an incompressible fluid incorporating the Boussinesq approximation:

\[ \frac{\partial \mathbf{\omega}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{\omega} - (2 \mathbf{\Omega} + \mathbf{\omega}) \cdot \nabla \mathbf{u} \]

\[ = \nabla \times g \beta T \frac{\omega}{\mu_0 \rho_0} \nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \nu \nabla^2 \mathbf{\omega}, \]

(7)

where \( \mathbf{\omega} \) is the vorticity, \( T \) is the total temperature, which is the sum of the basic state (conductive) temperature and the deviation from this state. If we consider slow and steady motions in an inviscid fluid and assume there are no body forces arising either from the magnetic field or from temperature (or density) perturbations, then we immediately obtain \( 2 \mathbf{\Omega} \mathbf{u}/\partial z = 0 \), the famous Proudman–Taylor theorem\(^{22,23} \). This axial invariance of velocity is also known as the *geostrophic* state, where the Coriolis force \( 2\mu_0 \Omega \times \mathbf{u} \) is in exact balance with the horizontal pressure gradient \( -\nabla p \) in eq. (4). As the flow is purely two-dimensional, it cannot transmit heat across the fluid layer. Evidently, the onset of convection can occur only if the Proudman–Taylor (or rotational) constraint is broken, which would be the case if viscous diffusion is present\(^24 \). The smaller the viscosity, the more difficult it is to start convection because the buoyancy forces must be large. In rotating convection the flow takes the form of rolls (Taylor columns) aligned with the axis of rotation. Later we shall look at the consequences of the Proudman–Taylor theorem for convection in the Earth’s core.

We now consider the case when the above fluid layer is electrically conducting and permeated by a magnetic field, \( \mathbf{B} \). The Lorentz force in eq. (7) overcomes the rotational constraint by inducing velocity gradients via the Coriolis force \( 2\mu_0 \mathbf{u}/\partial z \), a process that occurs even for zero viscosity. The effect of the magnetic field may also be understood from energy arguments. An axially varying azimuthal field causes axial variations in the lengthscale of the fluid columns perpendicular to \( \mathbf{\Omega} \), with regions in a strong field being preferentially thicker than regions in a weak field. Any increase in lateral dimension of the columns would result in reduced energy dissipation, so that buoyancy does not have to work so hard to maintain
convection. The above role of the magnetic field in aiding rotating convection is in contrast to its role in non-rotating fluids where the field tends to suppress motions by Ohmic dissipation. In summary, rotation tends to suppress convection, whereas the magnetic field makes it easier to set up convection in a rotating fluid.

Nonlinear convective geodynamo models

In a geodynamo model the fundamental MHD equations (3)–(6) are made dimensionless and then solved numerically for a Boussinesq fluid between two concentric spherical surfaces that mimic the Earth’s inner core boundary (ICB) and the core-mantle boundary (CMB). The ratio of inner to outer radius, \( r_i/r_o \) is usually chosen to be 0.35. The computational domain is shown schematically in Figure 2. The standard numerical method and boundary conditions have been discussed in previous papers. We begin by looking at the dimensionless parameters.

Dimensionless parameters

The basic dimensionless groups used in dynamo models are the Ekman number, the Rayleigh number, the Prandtl number, and the magnetic Prandtl number. (The magnetic Reynolds number, \( Rm = uL/\kappa \) is an intrinsic parameter.)

The Rayleigh number for convection can have different definitions depending on the mode of heating; for differential heating (Figure 2) \( Ra = g\beta\Delta T L^3/\nu \kappa \), where \( L \) is the gap-width of the spherical shell and \( \kappa \) the thermal diffusivity. In dynamo models the classical Rayleigh number is often multiplied by the Ekman number to give a ‘modified’ Rayleigh number, \( Ra_m = g\beta\Delta T L^2/2\Omega \kappa \). Estimates for \( Ra \) in the core vary from approximately the critical value for onset of convection, \( Ra_c \), to several orders of magnitude above \( Ra_c \) (ref 30, 31) even if the turbulent value of the diffusivity \( \kappa_T \) is adopted in place of its molecular value. The Prandtl number, \( Pr \) is given by \( \nu/\kappa \) and the magnetic Prandtl number, \( Pm \) is \( \nu/\eta \). The Roberts number, given by \( Q = PmPr^{-1} = \kappa/\eta \), is a popular dimensionless group in many models, with a molecular value of \( \sim 10^{-6} \) and a turbulent value of order unity.

Present-day numerical geodynamo models mostly operate in the parameter regime \( E \gtrsim 10^7 \), \( Pr \sim 1 \), \( Q = \kappa/\eta \sim 0.05 \) and \( Ra/Ra_c \leq 100 \).

Linear theory of rapidly rotating convection

The theory for the onset of convection under rapid rotation (in the low Ekman number limit) was originated nearly four decades ago and developed further recently. These analyses are linear in the sense that the nonlinear inertial term in the momentum equation is neglected. It has been shown that the critical wavenumber at onset of convection, \( m_c \) varies as \( E^{-1/3} \) and the critical Rayleigh number, \( Ra_c \) varies as \( E^{-4/3} \). (Here \( E \) is the Ekman number.) As \( E \) is lowered, the critical Rayleigh number increases and convection takes the form of several tall thin columns. In the limit \( E \to 0 \) (zero viscosity), \( Ra \) goes to infinity, implying that no convection can occur without viscosity. The value of \( Ra \), obtained from the theory of convection is generally accepted as the reference state in numerical dynamo models: The value of \( Ra/Ra_c \) tells us how strongly a dynamo is driven.

Although the magnetic field is known to reduce the value of \( Ra \), (see the section ‘Onset of convection and the effects of rotation and magnetic field’, above), it is not easy to evaluate the true value of \( Ra \) in a nonlinear dynamo.

Nonlinear dynamo models

Elsasser’s idea of solving the three-dimensional dynamo problem had to wait until the mid-1990s, when the first numerical solutions for the MHD equations appeared. As early models had no hope of realizing Earth-like parameters, they used hyperdiffusion to absorb the energy in higher spherical harmonics (small scales), but this...
approach led to unphysical effects such as a much reduced azimuthal wavenumber. Other models used a high Ekman number or limited resolution in longitude, both of which helped reduce computational effort. These drawbacks were overcome in subsequent studies as computer speed increased. A simple convective dynamo, running from a prescribed initial state for temperature and magnetic field, has been adopted as the benchmark against which all dynamo codes can be tested for accuracy. Dynamo models from the last decade have reproduced several Earth-like features like the dipolar magnetic field, secular variation and occasional field reversals.

Figure 3 shows snapshots of the flow and field produced in two dynamo simulations. The columnar structure of convection has origins in the Proudman–Taylor theorem for a rotating fluid; the number of columns increases with decreasing Ekman number, as predicted by linear theory. At low $E$ the magnetic field does not appear to thicken fluid columns, so the flow is essentially similar to that found in nonmagnetic convection. For $E = 5 \times 10^5$ a westward drift is noted for both the velocity and magnetic fields, consistent with observations of secular variation. The large-scale dipolar magnetic field appears to be generated in the fluid columns, but a higher Rayleigh number might expel flux from the tangent cylinder. Models with basal heating produce dipolar fields for a wide range of $Ra/Rac$, whereas those with uniform internal heating (used to mimic radioactive heat sources in the core) often produce non-dipolar fields as well. Some dynamo models favour the $\alpha - \omega$ dynamo cycle for field regeneration, whereas others favour an $\alpha^2$ mechanism, where the toroidal field is produced by helical fluid motion in convection rolls rather than by axial gradients in the azimuthal flow.

We shall now discuss how dynamo models have improved our understanding of core flows and magnetic fields.

**Thermal winds and the mode of tangent cylinder convection**

Temperature (density) perturbations in the Earth’s core give rise to fluid motion whose behaviour can be predicted by considering the curl of the momentum eq. (7) in spherical polar coordinates $(r, \theta, \phi)$, while retaining the conditions of slow and steady motion and negligible viscosity. For nonmagnetic convection,

$$2\Omega \frac{\partial u_\theta}{\partial z} = \frac{g \beta}{r} \frac{\partial T}{\partial \theta}$$

(8)

$$2\Omega \frac{\partial u_\phi}{\partial z} = -\frac{g \beta}{\sin \theta} \frac{\partial T}{\partial \phi}$$

(9)

The winds or currents implied by eqs (8) and (9) are known as the thermal wind, studied extensively in geophysical fluid dynamics. In the northern hemisphere of the Earth’s atmosphere, the temperature difference between the warm equatorial air and the cold Arctic air gives rise to a positive $\partial T/\partial \theta$, and the resulting jet stream that flows from west to east shortens flight times for aircraft travelling eastward. There is evidence from secular variation of the geomagnetic field that there are anticyclonic (westward) polar vortices in the core. The origin of these vortices could be a thermal wind caused by the polar regions in the core being warmer than the equatorial regions. Order-of-magnitude estimates show that even small latitudinal temperature variations ($\sim 10^3$ K) can produce the observed anticyclonic vortices via eq. (8).

The magnitude of the thermal wind in the Earth is thought to be radically affected by its self-generated magnetic field. To understand this we must look at convection within the tangent cylinder (TC), an imaginary cylinder that touches the solid inner core of radius 1220 km, about 0.35 times the radius of the whole fluid core (see Figure 2). The rapid rotation of the Earth’s core divides convection into two distinct regions, inside and outside the TC. Outside the TC convection occurs more readily than inside the TC because heat and composition can be convected outward by tall columns in which fluid motion...
is almost independent of the axial coordinate \( z \), in an approximately geostrophic balance between the Coriolis force and the pressure gradient. (The CMB prevents complete geostrophy.) Inside the TC heat transport from the ICB to the CMB requires axial \( (z) \) motions that vary appreciably in the \( z \)-direction as both the ICB and CMB are impenetrable\(^{44} \). Numerical simulations of rotating convection and dynamos confirm that the Rayleigh number required for the onset of convection is much higher inside the TC than outside. When convection occurs in the TC, the flow often takes the form of a single coherent plume (hot spot) that extends from the inner boundary right up to the polar region, but offset from the rotation axis\(^{50,42} \); see Figure 2. The plume does not remain at the same longitude, but migrates in a rather irregular fashion, but generally westward. In nonmagnetic convection, however, the flow in the TC takes the form of tall thin columns whose radius is controlled by viscosity. In comparison with the viscous mode of convection, the magnetic mode produces stronger polar vortices which are non-axisymmetric (Figure 2). Interestingly, when nonlinear inertial forces are not small in the momentum equation (see below), the polar vortices become cyclonic, which suggests that inertial forces must be small enough in the Earth’s core to make the vortices anticyclonic.

A couple of issues arise from the study of thermal winds. First, the variation of azimuthal velocity along \( z \) indicates not only that the flow may be anticyclonic at the poles, but also that it may be cyclonic near the ICB. This could result in a superrotation of the inner core relative to the mantle. There is some seismological evidence that the inner core is rotating faster than the mantle\(^{52-54} \), but this is not conclusive\(^{55} \). On the other hand, gravitational coupling between the inner core and the mantle could prevent free relative motion between the two\(^{56} \). Another issue is that the magnetic field intensity in the poles is weak, whereas the field is concentrated in high-latitude flux lobes outside the TC (see below). While this could mean that convection inside the TC is too weak to generate a substantial field, it is more likely that strong magnetic-mode convection in the form of a plume expels flux from the polar regions.

**How important is inertia for the Earth’s core?**

For large-scale motions in the Earth’s core, we can estimate from eq. (4) that \( |(\mathbf{u} \cdot \nabla)\mathbf{u}| = u_\alpha^2/L \), where \( L \) is the typical lengthscale of the core, which we define as the difference between the outer and inner core radii \( L = r_o - r_i = 2260 \) km, and \( u_\alpha \) a typical velocity of core motion. A velocity \( u_\alpha \sim 3 \times 10^{-4} \) is estimated from the observed secular variation based on the assumption that the field is frozen into the flow\(^3 \). (Here we neglect magnetic diffusion in the induction eq. (3) and invert for the flow velocity at the core surface.) The Rossby number, which is the ratio of inertial to Coriolis forces in eq. (4) is then,

\[
Ro = \frac{u_\alpha}{L \Omega} = 2 \times 10^{-6} \ll 1.
\]

Thus the flow in the core will be dominated by rotation even if the core velocities are somewhat larger than those observed near the CMB. It may be argued that the above argument underestimates the role of inertia if core convection takes the form of tall thin columns, as noted in Figure 3. If we suppose that the columns have extent \( \sim L \) parallel to the rotation axis \( z \), and a much shorter length \( L_\perp \) perpendicular to the axis, then the curl of eq. (4) gives

\[
|\nabla \times (\mathbf{u} \cdot \nabla)\mathbf{u}| = \frac{u_\alpha^2}{L_\perp}, \quad |(\mathbf{\Omega} \cdot \nabla)\mathbf{u}| = \frac{\Omega u_\alpha}{L_\perp},
\]

which gives a balance between the inertial and Coriolis terms when

\[
L_\perp = L_R = \left( \frac{u_\alpha L}{\Omega} \right)^{1/2} \sim 4 \text{ km}
\]

in the core. Here \( L_R \) is the Rhines length\(^57 \). Motions on this scale are unlikely to be relevant to the dynamo process as magnetic fields on this lengthscale will decay in less than a year, assuming a value of \( 2 \text{ m}^2\text{s}^{-1} \) for the magnetic diffusivity\(^58 \). Those columns that have had their horizontal scale enhanced by the effect of the magnetic field so that \( L_\perp \gg L_R \) would be important for the geodynamo.

Numerical dynamo models can have inertia in them because it is not possible to solve the dynamo at realistic values of the Ekman number, \( E \) and magnetic Prandtl number, \( Pm \). The Rossby number may be expressed as \( Ro = EPm^{-1}Rm \). Dynamo models giving \( Rm \sim 100 \) frequently use \( Pr = Pm = 1 \) and \( E \sim 10^{-3} \), so \( Ro \sim 10^{-2} \). Recalling that the inertial force over a column can be enhanced significantly, the local value of the Rossby number can be much higher. Furthermore, if a low value of \( Pm \) is put into a dynamo code\(^56,59 \) Ro can easily be of order unity. It must be emphasized that these large values of \( Ro \) arise only because \( E \) has to be enhanced for numerical stability. A low value of \( E \) does allow a low value of \( Pm \) without increasing the magnitude of inertia\(^60,66 \). (Also see the parameters for Figure 3b.) Sreenivasan and Jones\(^64 \) varied the Prandtl numbers \( Pr \) and \( Pm \) leaving other parameters fixed. For large \( Pr \approx Pm \) they obtained a low-inertia solution for numerically accessible Ekman number. In this regime the dynamo is dipolar and the principal force balance is between the magnetic, Archimedean (buoyancy) and Coriolis forces, also known as the MAC balance. (For \( E \sim 10^{-4} \) viscous forces are small except in the boundary layers where the velocity gradients are appreciable.) The MAC balance was envisaged for the geodynamo by Taylor\(^61 \) and Braginsky\(^62 \). As
In the magnetic field: If the mantle were perfectly spherically symmetric, the core flow would be free to evolve relative to it, eliminating the possibility of preferred longitudes. Hide\textsuperscript{72} was the first to suggest that the lower-mantle variations cast their signature on the morphology of the geomagnetic field. Cold regions in the lower mantle could cause preferential cooling of the core, downwelling, and concentration of vertical magnetic flux at the core surface. This qualitative suggestion has now been explored in many convection and dynamo models, mostly by imposing a thermal boundary condition with the same structure as a ‘tomographic’ model of shear wave velocity variation in the lowermost mantle (Figure 4\textit{a}). The dominant pattern is a fast (cold) ring around the Pacific rim with slow (hot) regions beneath the Pacific and Africa. The largest term in a spherical harmonic expansion of the tomography is $Y_2^2$, and many studies have simplified the boundary condition to this equatorially symmetric harmonic.

Early numerical simulations of core–mantle interaction were on nonmagnetic convection with infinite Prandtl number ($Pr$) and laterally varying temperature boundary conditions. At slightly supercritical Rayleigh number the drifting pattern of fluid rolls becomes stationary, or ‘locked’ to the boundary, provided the lateral variation in boundary heating is sufficiently strong. It was suggested that locking occurs when the wavelength of convection with homogeneous boundary conditions is similar to the wavelength of the boundary anomalies\textsuperscript{73}. Fully self-consistent geodynamo simulations with inhomogeneous thermal boundary conditions have since been used to explore boundary effects on the frequency of field reversals\textsuperscript{74}, secular variation of the geomagnetic field\textsuperscript{75}, the time-averaged magnetic field\textsuperscript{76} and core surface flows\textsuperscript{77}. These studies generally support the idea that lower-mantle shear wave velocity correlates with some aspects of the time-averaged field, but there is little evidence of any simple boundary locking in any of the results, nor is there
any direct similarity between snapshots of the solutions and the present-day geomagnetic field. A dynamo solution in which the magnetic field was locked to the boundary anomalies defined by seismic tomography was obtained recently. The solution was not stationary, but the characteristic four main lobes persisted for many diffusion times at the same sites as the main lobes of the geomagnetic field (Figure 4c). However, the parameters at which boundary locking was obtained were not Earth-like: The Rayleigh number for convection had to be set to a low, marginally supercritical value; consequently the magnetic diffusivity in the model had to be kept to a tenth of the thermal diffusivity \((q = \kappa/\eta \sim 10)\) to make a dynamo possible. To understand why a weakly convective parameter regime is crucial for locking, one must return to the thermal wind balance,

\[
2\Omega \frac{\partial u}{\partial z} = \nabla \times [g \beta T],
\]

whose horizontal components were given by eqs (8) and (9). With boundary anomalies, the right-hand side of eq. (13) is made up of two parts, the temperature gradients produced by free convection and the temperature gradients originating from non-axisymmetric lateral variations at the boundary. Locking is obtained when the boundary-driven variations dominate the force balance. (Also see Figure 5.) Equation (13) is significant in that it allows fluid motion of any wavenumber to be produced by a prescribed thermal variation at the boundary. Previous studies had suggested that locking occurs when the wavenumber of convection is similar to the wavenumber of the boundary anomalies. Clearly, no such matching of wavenumbers is required for locking produced by eq. (13) because it does not rely on convection in the first place!

How is locking affected by a high convective Rayleigh number and a low Ekman number? If the free convection-driven temperature gradients are stronger than the boundary-driven gradients in eq. (13), the velocity field is decoupled from the boundary and free to drift azimuthally. For low Ekman numbers locking looks progressively difficult. Since the critical Rayleigh number for onset of nonmagnetic convection increases with decreasing Ekman number \((Ra_c - E^{-4/3})\), even a marginally supercritical convective state can generate strong thermal winds that swamp the boundary variations. The above arguments led to a new model, where a combination of bottom heating and a uniform heat sink made convection much weaker at the top than at the bottom. As boundary anomalies were allowed to dominate the thermal wind balance in the upper regions, partial locking of the flow and magnetic field was obtained. This model also obviated the need for a small magnetic diffusivity for dynamo action.

Recent studies on locked dynamos have produced a few surprising results. First, the force balance is different from that in convection-driven dynamos. As the boundary-driven thermal wind balance is enforced, a secondary balance between the Lorentz and inertial forces follows, yielding an equipartition \((u = B)\) solution. That is, the kinetic and magnetic energies are equal, and the magnetic field and velocity field are tied together to one lengthscale. Although the Earth’s magnetic field is not rigidly locked to lower-mantle variations, the above result does indicate that core–mantle coupling can place a constraint on the ratio of kinetic to magnetic energies. Secondly, lateral variations by themselves can, under small background convection, generate a magnetic field whose structure is fixed by the boundary wavenumber. This result suggests that lower-mantle variations might play a role in field generation.

**Modelling geomagnetic field reversals**

Geomagnetic reversals are perhaps the most interesting phenomena in geophysics, and perhaps the least understood. Magnetic records from ancient volcanic rock and sediments are our main source of information on reversals. The average reversal frequency in the last 20 Ma was about 5 every Ma, but the last reversal had occurred 0.78 Ma ago. From about 118–83 Ma ago, a period known as the Cretaceous Superchron, there are no recorded reversals of the field. This period seems to show a significantly reduced secular variation, leading to speculation that the geodynamo was passing through a relatively stable phase. During reversals the axial dipole moment can decrease by a factor of 5 compared to its time-averaged value, and curiously, the dipole moment begins its decline 60–80 kyr before a reversal and recovers.
rapidly (within a few thousand years) after the dipole transition. The geomagnetic dipole moment has been decreasing at a rapid rate in recent history, so the Earth might be in the early stage of a reversal.

The first polarity reversal in a geodynamo model was obtained by Glatzmaier and Roberts. Since then several strongly driven dynamos have reported spontaneous polarity reversals, some reminiscent of paleomagnetic reversals. Except for one model, most reversing dynamos have operated in a high Ekman number ($E \geq 10^{-4}$) regime, which has been justified on grounds of simplicity and suitability for long simulations. A high Ekman number is a natural choice for studying reversals as a strong convective state is realized for moderate Rayleigh numbers. A high-$E$, $Pm \sim 1$ dynamo has a Rossby number several orders of magnitude higher than that in the Earth, although it is claimed that the Rossby number based on the typical lengthscale of convection may not be far from the Earth’s value. As the magnetic Reynolds number for flows of the Rhines lengthscale is only $\sim 1$, it is not clear that buoyancy will replenish vortices that are rapidly damped by the magnetic field.

Although the sequence of events during a dynamo reversal has been studied in several numerical models, the fundamental cause of field reversals is a mystery. An insight into departures from dipolar symmetry, including reversals, could perhaps be obtained by addressing the question of why rapidly rotating dynamos have a preference for dipolar solutions. We shall discuss this briefly in the concluding section.

The future of geodynamo modelling

Geodynamo models operating in vastly different parameter regimes have been successful in reproducing the main features of the geomagnetic field, the most important being the large-scale dipolar structure itself. This has provided the impetus to explore new dynamical regimes which are hopefully more Earth-like than previous models. The computationally difficult parameter space of low Ekman number and low Roberts number is explored on the supposition that it would be a better representation of core convection, but dynamos in this regime have not produced magnetic fields that look like the Earth. A fundamental study of the rapidly rotating regime should however be welcomed. Rotation with concomitant helical fluid motions in columns has been thought to produce dipolar fields, but recent studies suggest that magnetic field-induced flows can explain the preference for dipolar fields over quadrupolar fields (Sreenivasan and Jones, work in progress). These studies reaffirm our faith in nonlinear dynamos where the back-reaction of the magnetic field on the flow through the Lorentz force is given the importance it deserves. As we see below, they might also offer an insight into magnetic field reversals.

As convection in the Earth’s core might take the form of tall columns parallel to the rotation axis, it makes sense to consider the axial component of the MAC force balance (see sub-section ‘How important is inertia for the Earth’s core?’)

$$2\Omega \partial u_z/\partial z + g\beta \nabla \times (\mathbf{v}_r \cdot \mathbf{z}) + \nabla \times [(\nabla \times \mathbf{B}) \mathbf{z}] = 0.$$  

This effect is manifest in Figure 6, where the Lorentz force enhances the axial kinetic energy in localized streaks. The growth in axial velocity contributes to an increase in helicity (the dot product of velocity and vorticity), an important quantity for dynamo action. The field-induced flow is much weaker for a dynamo with a quadrupolar field because both damping and driving forces peak at the equator. We therefore have a mechanism giving strong preference for dipolar fields over quadrupolar fields. For a field reversal to occur, it is perhaps necessary to be in a strongly convectiong regime, where

![Figure 6](image-url)

**Figure 6.** Azimuthally averaged meridional plots for a dynamo model at $E = 5 \times 10^{-5}$, $Ra/Ra_c = 6$, $Pr = 1$ and $q = Pm = 1$. Positive values are shown in red and negative values in blue. (a) Azimuthal magnetic field, $B_a$. (b) Axial kinetic energy density, $\frac{1}{2} u_z^2$. No-slip, isothermal and electrically insulating boundary conditions are used. This model produces a stable, dipolar magnetic field.
the buoyancy force in regions away from the equator is large enough to cancel the effect of the Lorentz force, so that the production of $\partial u_r/\partial z$ in eq. (14) is inhibited. This mechanism of breakdown of dipolar symmetry must be tested in future dynamo models.

Dynamo models are a powerful tool for testing various other hypotheses for the Earth’s core. For example, the possibility of driving a dynamo at least in part by lateral variations in the lower mantle suggests that the Rayleigh number for convection need not be high. A related issue is whether convection occurs everywhere in the core. We know from simulations that polarity reversals are realizable only in strongly driven dynamos. On the other hand, persistent core–mantle coupling through the boundary-driven thermal wind can only be obtained when convection is small. These two extreme regimes could coexist in a stably stratified model where convection is strong at depth but boundary anomalies control fluid motion in the upper regions, a scenario consistent with independent arguments based on compositional buoyancy. It is a matter of concern that many models might be using unphysical basic state buoyancy profiles, prescribing either uniform heat flux throughout the core, or even worse, heat flux that increases from the CMB to the CMB.

Geodynamo models will be called upon in future to alleviate the deficiencies in our understanding of secular variation, field reversals, torsional oscillations and lower-mantle effects. Our understanding of the Earth’s dynamo is far from complete, but with improved geophysical data from satellite missions and insights from laboratory experiments we can hope that newer models will emerge to provide useful comparisons with the observed geomagnetic field.